

STRANGE GOINGS ON IN QUARK MATTER

THOMAS SCHÄFER

Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800 and Riken-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973

We review recent work on how the superfluid state of three flavor quark matter is affected by non-zero quark masses and chemical potentials.

1 Introduction

The study of hadronic matter at high baryon density has recently attracted a lot of interest. At zero baryon density chiral symmetry is broken by a quark-anti-quark condensate. At high density condensation in the quark-anti-quark channel is suppressed. Instead, attractive interactions in the color anti-symmetric quark-quark channel favor the formation of diquark condensates. As a consequence, cold dense quark matter is expected to be a color superconductor^{1,2,3}. The symmetry breaking pattern depends on the density, the number of quark flavors, and their masses. A particularly symmetric phase is the color-flavor-locked (CFL) phase of three flavor quark matter⁴. This phase is believed to be the true ground state of ordinary matter at very large density^{5,6,7}.

The CFL phase is characterized by the order parameter

$$\langle q_{L,i}^a C q_{L,j}^b \rangle = -\langle q_{R,i}^a C q_{R,j}^b \rangle = \phi (\delta_i^a \delta_j^b - \delta_i^b \delta_j^a). \quad (1)$$

This order parameter breaks both the global $SU(3)_L \times SU(3)_R \times U(1)_V$ flavor symmetry and the local $SU(3)_C$ color symmetry of QCD. As a result, all fermions are gapped and all gluons acquire a mass via the Higgs mechanism. Color-flavor-locking leaves a vectorial $SU(3)_V$ unbroken. This symmetry is the diagonal subgroup of the original $SU(3)_L \times SU(3)_R \times SU(3)_C$ symmetry. This means that the color-flavor-locked phase exhibits the chiral symmetry breaking patterns $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$, just like QCD at zero baryon density. However, the mechanism of chiral symmetry breaking is quite unusual. The primary order parameter (1) does not couple left and right-handed quarks. Chiral symmetry is broken because both left and right-handed flavor are “locked” to color, and because color is a vectorial symmetry.

At baryon densities relevant to astrophysical objects distortions of the pure CFL state due to non-zero quark masses are probably

important^{8,9,10,11,12}. In the present work we wish to study this problem using the effective chiral theory of the CFL phase¹³ (CFL χ Th).

2 CFL Chiral Theory (CFL χ Th)

For excitation energies smaller than the gap the only relevant degrees of freedom are the Goldstone modes associated with the breaking of chiral symmetry and baryon number. The interaction of the Goldstone modes is described by an effective Lagrangian of the form¹³

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{f_\pi^2}{4} \text{Tr} [\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v_\pi^2 \partial_i \Sigma \partial_i \Sigma^\dagger] + [A_1 \text{Tr}(M \Sigma^\dagger) \text{Tr}(M \Sigma^\dagger) \\ & + A_2 \text{Tr}(M \Sigma^\dagger M \Sigma^\dagger) + A_3 \text{Tr}(M \Sigma^\dagger) \text{Tr}(M^\dagger \Sigma) + h.c.] + \dots \end{aligned} \quad (2)$$

Here $\Sigma = \exp(i\phi^a \lambda^a / f_\pi)$ is the chiral field and f_π is the pion decay constant. We have suppressed the singlet fields associated with the breaking of the exact $U(1)_V$ and approximate $U(1)_A$ symmetries. The theory (2) looks superficially like ordinary chiral perturbation theory. There are, however, some important differences. Lorentz invariance is broken and Goldstone modes travel with the velocity $v_\pi < c$. In the CFL phase the ordinary chiral condensate $\langle \bar{\psi} \psi \rangle$ is small and the dominant order parameter for chiral symmetry breaking is $\langle (\bar{\psi} \psi)^2 \rangle$. As a consequence, the coefficient of $\text{Tr}(M \Sigma)$ is exponentially small and the leading mass terms are quadratic in M .

The pion decay constant f_π and the coefficients $A_{1,2,3}$ can be determined using matching techniques. Matching expresses the requirement that Green functions in the effective chiral theory and the underlying microscopic theory, QCD, agree. The pion decay constant is most easily determined by coupling gauge fields $W_{L,R}$ to the left and right flavor currents. As usual, this amounts to replacing ordinary derivatives by covariant derivatives. The time component of the covariant derivative is given by $\nabla_0 \Sigma = \partial_0 \Sigma + iW_L \Sigma - i\Sigma W_R$ where we have suppressed the vector index of the gauge fields. In the CFL vacuum $\Sigma = 1$ the axial gauge field $W_L - W_R$ acquires a mass by the Higgs mechanism. From (2) we get

$$\mathcal{L} = \frac{f_\pi^2}{4} \frac{1}{2} (W_L - W_R)^2. \quad (3)$$

The coefficients $A_{1,2,3}$ can be determined by computing the shift in the vacuum energy due to non-zero quark masses in both the chiral theory and the microscopic theory. In the chiral theory we have

$$\Delta \mathcal{E} = -[A_1 (\text{Tr}(M))^2 + A_2 \text{Tr}(M^2) + A_3 \text{Tr}(M) \text{Tr}(M^\dagger) + h.c.]. \quad (4)$$

3 High Density Effective Theory (HDET)

In this section we shall determine the mass of the gauge field and the shift in the vacuum energy in the CFL phase of QCD at large baryon density. This is possible because asymptotic freedom guarantees that the effective coupling is weak. The QCD Lagrangian in the presence of a chemical potential is given by

$$\mathcal{L} = \bar{\psi}(i\not{D} + \mu\gamma_0)\psi - \bar{\psi}_L M \psi_R - \bar{\psi}_R M^\dagger \psi_L - \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a, \quad (5)$$

where M is a complex quark mass matrix which transforms as $M \rightarrow L M R^\dagger$ under chiral transformations $(L, R) \in SU(3)_L \times SU(3)_R$ and μ is the baryon chemical potential. If the baryon density is very large perturbative QCD calculations can be further simplified. The main observation is that the relevant degrees of freedom are particle and hole excitations in the vicinity of the Fermi surface. We shall describe these excitations in terms of the field $\psi_+(\vec{v}_F, x)$, where \vec{v}_F is the Fermi velocity. At tree level, the quark field ψ can be decomposed as $\psi = \psi_+ + \psi_-$ where $\psi_\pm = \frac{1}{2}(1 \pm \vec{\alpha} \cdot \vec{v}_F)\psi$. Integrating out the ψ_- field at leading order in $1/p_F$ we get^{14,15,16}

$$\begin{aligned} \mathcal{L} = & \psi_{L+}^\dagger (iv \cdot D) \psi_{L+} - \frac{\Delta}{2} \left(\psi_{L+}^{ai} C \psi_{L+}^{bj} (\delta_{ai} \delta_{bj} - \delta_{aj} \delta_{bi}) + \text{h.c.} \right) \\ & - \frac{1}{2p_F} \psi_{L+}^\dagger ((\not{D}_\perp)^2 + M M^\dagger) \psi_{L+} + (R \leftrightarrow L, M \leftrightarrow M^\dagger) + \dots, \end{aligned} \quad (6)$$

where $D_\mu = \partial_\mu + igA_\mu$, $v_\mu = (1, \vec{v})$ and i, j, \dots and a, b, \dots denote flavor and color indices. The longitudinal and transverse components of γ_μ are defined by $(\gamma_0, \vec{\gamma})_\parallel = (\gamma_0, \vec{v}(\vec{\gamma} \cdot \vec{v}))$ and $(\gamma_\mu)_\perp = \gamma_\mu - (\gamma_\mu)_\parallel$. In order to perform perturbative calculations in the superconducting phase we have added a tree level gap term $\psi_{L,R} C \Delta \psi_{L,R}$.

The mass of a flavor gauge field can be determined by computing the corresponding polarization function in the limit $q_0 = 0, \vec{q} \rightarrow 0$. We find $\Pi_{00}^{LL} = \Pi_{00}^{RR} = -\Pi_{00}^{LR} = m_D^2/4$ with $m_D^2 = (21 - 8\log(2))p_F^2/(36\pi^2)$. Matching against equ. (3) we get¹⁷

$$f_\pi^2 = \frac{21 - 8\log(2)}{18} \left(\frac{p_F^2}{2\pi^2} \right). \quad (7)$$

Our next task is to compute the mass dependence of the vacuum energy. To leading order in $1/p_F$ there is only one operator in the high density effective theory

$$\mathcal{L} = -\frac{1}{2p_F} \left(\psi_{L+}^\dagger M M^\dagger \psi_{L+} + \psi_{R+}^\dagger M^\dagger M \psi_{R+} \right). \quad (8)$$

This term arises from expanding the kinetic energy of a massive fermion around $p = p_F$. We note that $MM^\dagger/(2p_F)$ and $M^\dagger M/(2p_F)$ act like effective chemical potentials for left and right-handed fermions, respectively. Indeed, to leading order in the $1/p_F$ expansion, the Lagrangian (6) is invariant under a time dependent flavor symmetry $\psi_L \rightarrow L(t)\psi_L$, $\psi_R \rightarrow R(t)\psi_R$ where $X_L = MM^\dagger/(2p_F)$ and $X_R = M^\dagger M/(2p_F)$ transform as left and right-handed flavor gauge fields. If we impose this approximate gauge symmetry on the CFL chiral theory we have to include the effective chemical potentials $X_{L,R}$ in the covariant derivative of the chiral field,

$$\nabla_0 \Sigma = \partial_0 \Sigma + i \left(\frac{MM^\dagger}{2p_F} \right) \Sigma - i \Sigma \left(\frac{M^\dagger M}{2p_F} \right). \quad (9)$$

X_L and X_R contribute to the vacuum energy at $O(M^4)$

$$\Delta \mathcal{E} = \frac{f_\pi^2}{8p_F^2} \text{Tr} [(MM^\dagger)(M^\dagger M) - (MM^\dagger)^2]. \quad (10)$$

This result can also be derived directly in the microscopic theory¹¹. This means that we do not have to rely on the effective gauge symmetry in order to derive (9). $O(M^2)$ terms in the vacuum energy are generated by terms in the high density effective theory that are higher order in the $1/p_F$ expansion. We recently argued that these terms can be determined by computing chirality violating quark-quark scattering amplitudes in QCD¹⁸. At leading order in the $1/p_F$ expansion the chirality violating scattering amplitude can be represented as an effective four-fermion operator

$$\begin{aligned} \mathcal{L} = & \frac{g^2}{8p_F^4} \left((\psi_L^{A\dagger} C \psi_L^{B\dagger})(\psi_R^C C \psi_R^D) \Gamma^{ABCD} + (\psi_L^{A\dagger} \psi_L^B)(\psi_R^{C\dagger} \psi_R^D) \tilde{\Gamma}^{ACBD} \right) \\ & + (L \leftrightarrow R, M \leftrightarrow M^\dagger). \end{aligned} \quad (11)$$

Here, we have introduced the CFL eigenstates ψ^A defined by $\psi_i^a = \psi^A(\lambda^A)_{ai}/\sqrt{2}$, $A = 0, \dots, 8$. The tensors Γ is defined by

$$\begin{aligned} \Gamma^{ABCD} = & \frac{1}{8} \left\{ \text{Tr} [\lambda^A M (\lambda^D)^T \lambda^B M (\lambda^C)^T] \right. \\ & \left. - \frac{1}{3} \text{Tr} [\lambda^A M (\lambda^D)^T] \text{Tr} [\lambda^B M (\lambda^C)^T] \right\}. \end{aligned} \quad (12)$$

The tensor $\tilde{\Gamma}$ involves both M and M^\dagger and only contributes to field independent terms $\text{Tr}[MM^\dagger]$ in the vacuum energy. We can now compute the shift in the vacuum energy due to the effective vertex (11). The result

$$\Delta \mathcal{E} = -\frac{3\Delta^2}{4\pi^2} \left\{ \left(\text{Tr}[M] \right)^2 - \text{Tr}[M^2] \right\} + (M \leftrightarrow M^\dagger) \quad (13)$$

determines the coefficients $A_{1,2,3}$ in the CFL chiral theory. We find

$$A_1 = -A_2 = \frac{3\Delta^2}{4\pi^2}, \quad A_3 = 0, \quad (14)$$

which agrees with the result of Son and Stephanov¹⁷.

4 Kaon Condensation

Using the results discussed in the previous section we can compute the masses of Goldstone bosons in the CFL phase. We have argued that the expansion parameter in the chiral expansion of the Goldstone boson masses is¹¹ $\delta = m^2/(p_F\Delta)$. The first term in this expansion comes from the $O(M^2)$ term in (2), but the coefficients A contain the additional small parameter $\epsilon = (\Delta/p_F)$. In a combined expansion in δ and ϵ the $O(\epsilon\delta)$ mass term and the $O(\delta^2)$ chemical potential term appear at the same order. At this order, the masses of the flavored Goldstone bosons are

$$\begin{aligned} m_{\pi^\pm} &= \mp \frac{m_d^2 - m_u^2}{2p_F} + \left[\frac{4A}{f_\pi^2} (m_u + m_d) m_s \right]^{1/2}, \\ m_{K^\pm} &= \mp \frac{m_s^2 - m_u^2}{2p_F} + \left[\frac{4A}{f_\pi^2} m_d (m_u + m_s) \right]^{1/2}, \\ m_{K^0, \bar{K}^0} &= \mp \frac{m_s^2 - m_d^2}{2p_F} + \left[\frac{4A}{f_\pi^2} m_u (m_d + m_s) \right]^{1/2}. \end{aligned} \quad (15)$$

We observe that the pion masses are not strongly affected by the effective chemical potential but the masses of the K^+ and K^0 are substantially lowered while the K^- and \bar{K}^0 are pushed up. As a result the K^+ and K^0 meson become massless if $m_s \sim m_{u,d}^{1/3} \Delta^{2/3}$. For larger values of m_s the kaon modes are unstable, signaling the formation of a kaon condensate.

Once kaon condensation occurs the ground state is reorganized. For simplicity, we consider the case of exact isospin symmetry $m_u = m_d \equiv m$. The most general ansatz for a kaon condensed ground state is given by

$$\begin{aligned} \Sigma = \exp & \left(i\alpha [\cos(\theta_1)\lambda_4 + \sin(\theta_1)\cos(\theta_2)\lambda_5 \right. \\ & \left. + \sin(\theta_1)\sin(\theta_2)\cos(\phi)\lambda_6 + \sin(\theta_1)\sin(\theta_2)\sin(\phi)\lambda_7] \right). \end{aligned} \quad (16)$$

With this ansatz the vacuum energy is given by

$$V(\alpha) = -f_\pi^2 \left(\frac{1}{2} \left(\frac{m_s^2 - m^2}{2p_F} \right)^2 \sin(\alpha)^2 + (m_K^0)^2 (\cos(\alpha) - 1) \right), \quad (17)$$

where $(m_K^0)^2 = (4A/f_\pi^2)m_{u,d}(m_{u,d} + m_s)$ is the $O(M^2)$ kaon mass in the limit of exact isospin symmetry. Minimizing the vacuum energy we obtain $\alpha = 0$ if $m_s^2/(2p_F) < m_K^0$ and $\cos(\alpha) = (m_K^0)^2/\mu_{eff}^2$ with $\mu_{eff} = m_s^2/(2p_F)$ if $\mu_{eff} > m_K^0$. We observe that the vacuum energy is independent of θ_1, θ_2, ϕ . The hypercharge density is given by

$$n_Y = f_\pi^2 \mu_{eff} \left(1 - \frac{m_K^4}{\mu_{eff}^4} \right), \quad (18)$$

where $\mu_{eff} = m_s^2/(2p_F)$. We observe that within the range of validity of the effective theory, $\mu_{eff} < \Delta$, the hypercharge density satisfies $n_Y < \Delta p_F^2/(2\pi)$. The upper bound on the hypercharge density in the condensate is equal to the particle density contained within a strip of width Δ around the Fermi surface.

The symmetry breaking pattern is $SU(2)_I \times U(1)_Y \rightarrow U(1)$ where I is isospin and Y is hypercharge. This corresponds to three broken generators. However, there are only two Goldstone modes, the K^0 and the K^+ . This mismatch is related to the dispersion relations of the Goldstone modes^{19,20}. The K^0 is a standard Goldstone mode with $\omega \sim p$ whereas the K^+ is an anomalous mode with $\omega \sim p^2$. As explained in more detail in²⁰ the appearance of an anomalous Goldstone mode is related to the fact that the kaon condensed groundstate has a non-zero expectation value of isospin.

5 Summary

We have studied the groundstate of CFL quark matter for non-zero quark masses. We have argued that there is a new scale $m_s^2/(2p_F) \sim \sqrt{m_{u,d}m_s}(\Delta/p_F)$ which corresponds to the onset of kaon condensation. For $m_s^2/(2p_F) \sim 1$ CFL pairing breaks down completely. These results can be established using just dimensional analysis. If perturbation theory is reliable we can be more quantitative. To leading order in g , the critical strange quark mass for kaon condensation is

$$m_s|_{crit} = 3.03 \cdot m_d^{1/3} \Delta^{2/3}. \quad (19)$$

This result suggests that for values of the strange quark mass and the gap that are relevant to compact stars CFL matter is likely to support a kaon condensate.

It is instructive to compare kaon condensation in CFL matter with the kaon condensate discussed by Kaplan and Nelson²¹. Kaplan and Nelson suggested kaon condensation as a path from ordinary hadronic matter, which has a deficit of strange quarks, to strange quark matter. Ideal CFL matter

has exactly equal numbers of up, down and strange quarks²². However, if the strange quark mass is non-zero then ideal CFL matter is oversaturated in strangeness. CFL kaon condensation is a way for CFL matter to reduce its strangeness content.

Acknowledgements: I am grateful to P. Bedaque, D. T. Son, M. A. Stephanov, D. Toublan and J. J. Verbaarschot for fruitful collaboration. This work was supported in part by US DOE grant DE-FG-88ER40388. A different scenario for chiral vacuum alignment in the CFL phase was discussed by Lee²³.

U.S.D.O.E.-DE-ACO2-98CH10886

References

1. D. Bailin and A. Love, Phys. Rept. **107**, 325 (1984).
2. M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. **B422**, 247 (1998).
3. R. Rapp, T. Schäfer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).
4. M. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
5. T. Schäfer and F. Wilczek, Phys. Rev. Lett. **82**, 3956 (1999).
6. T. Schäfer, Nucl. Phys. **B575**, 269 (2000).
7. N. Evans, J. Hormuzdiar, S. D. Hsu and M. Schwetz, Nucl. Phys. B **581**, 391 (2000).
8. M. Alford, J. Berges and K. Rajagopal, Nucl. Phys. **B558**, 219 (1999).
9. T. Schäfer and F. Wilczek, Phys. Rev. **D60**, 074014 (1999).
10. T. Schäfer, Phys. Rev. Lett. **85**, 5531 (2000).
11. P. F. Bedaque and T. Schäfer, hep-ph/0105150.
12. D. B. Kaplan and S. Reddy, hep-ph/0107265.
13. R. Casalbuoni and D. Gatto, Phys. Lett. **B464**, 111 (1999).
14. D. K. Hong, Phys. Lett. B **473**, 118 (2000).
15. D. K. Hong, Nucl. Phys. B **582**, 451 (2000).
16. S. R. Beane, P. F. Bedaque and M. J. Savage, Phys. Lett. B **483**, 131 (2000).
17. D. T. Son and M. Stephanov, Phys. Rev. **D61**, 074012 (2000), erratum: hep-ph/0004095.
18. T. Schäfer, hep-ph/0109052.
19. V. A. Miransky and I. A. Shovkovy, hep-ph/0108178.
20. T. Schäfer, D. T. Son, M. A. Stephanov, D. Toublan and J. J. Verbaarschot, hep-ph/0108210.
21. D. B. Kaplan and A. E. Nelson, Phys. Lett. **B175**, 57 (1986).
22. K. Rajagopal and F. Wilczek, Phys. Rev. Lett. **86**, 3492 (2001).
23. T. Lee, Phys. Lett. B **503**, 307 (2001).